

## **Problemath Series 2**

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(Deadline: Monday 15 November 14:00)

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### **Problemath 4**

While negotiating for the formation of a new (Belgian) government, 30 negotiators will sit around a round table, each at an equal distance from his two neighbours; 30 boxes, each carrying the name of a negotiator have been placed in advanced in front of each of the seats, on the table.

On the first day, everyone is so confused that no negotiator sits in front of the box bearing his name. The “formator”, the person in charge of creating the government, maintains that if such a situation happens, it is always possible to rotate the table so that at least two negotiators sit in front of their names.

Is such a statement correct?

### **Problemath 5**

Is there a polynomial  $p(x)$ , with integer coefficients, and three distinct integers  $a, b$  and  $c$  no two of

which are the same, such that  $p(a) = b$ ,  $p(b) = c$  and  $p(c) = a$ ?

## **Problemath 6**

*Is there a black hole in  $\mathbb{N}$ ?*

Starting from any natural number  $n$  (written in the decimal system), one builds the number  $\alpha(n)$  by writing in a row the number of even digits, the number of odd digits and the total number of digits of  $n$ . For instance 80322057626942 has 10 even digits, 4 odd digits and 14 digits altogether, so  $\alpha(n) = 10414$ . By iterating this process, one obtains the sequence

80322057626942, 10414, 325, 123, 123, 123, ... . Here is another example: 5771, 044, 303, 123, 123, ... .

Is any natural number inexorably attracted by 123, in other words, is any sequence of the form

$n, \alpha(n), \alpha^2(n), \alpha^3(n), \dots$  where  $i$  in the  $\alpha^i(n)$  is not an exponent, but refers to the iterated term of the sequence, constant (and equal to 123), starting from a certain term?